Measuring the Foaminess of Space-Time with Gravity-Wave Interferometers

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'T was noted in heaven, 't was felt in hell,

And echo caught faintly the noise as it fell...

(Slightly modified from *"Enigma. The letter H"* by C.M. Fanshawe)

Abstract

By analyzing a gedanken experiment designed to measure the distance l between two spatially separated points, we find that this distance cannot be measured with uncertainty less than $(ll_P^2)^{1/3}$, considerably larger than the Planck scale l_P (or the string scale in string theories), the conventional-wisdom uncertainty in distance measurements. This limitation to space-time measurements is interpreted as resulting from quantum fluctuations of space-time itself. Thus, at very short distance scales, space-time is "foamy." This intrinsic foaminess of space-time provides another source of noise in the interferometers. The LIGO/VIRGO and LISA generations of gravity-wave interferometers, through future refinements, are expected to reach displacement noise levels low enough to test our proposed degree of foaminess in the structure of space-time. We also point out a simple connection to the holographic

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principle which asserts that the number of degrees of freedom of a region of

space is bounded by the area of the region in Planck units.

I. INTRODUCTION

Quantum mechanics and general relativity, the two pillars of modern physics, are very useful in describing the phenomena in their respective domains of physics. Unfortunately, their synthesis has been considerably less successful. It is better known for producing a plethora of puzzles from the embarrassing cosmological constant problem [1] to the enigma of possible information loss associated with black hole evaporation [2]. String theory is a reaction to this crisis. Nowadays, it is the main contender to be the microscopic theory of quantum gravity. But even without the correct theory of quantum gravity (be it string theory or something else), we know enough about quantum mechanics and gravity to study its low-energy limit. In particular, we would like to know what that limit of quantum gravity can tell us about the structure of space-time. In this article, we will combine the general principles of quantum mechanics with those of general relativity to address the problem of quantum measurements of space-time distances.

But first, let us recall what quantum mechanics and general relativity have to say about the nature of space-time distance measurements. In quantum mechanics, we specify a space-time point simply by its coordinates; hardly do we feel the need to give a prescription to spell out how the coordinates are to be measured. This lax attitude will not do with general relativity. According to general relativity, coordinates do not have any meaning independent of observations; in fact, a coordinate system is defined only by explicitly carrying out space-time distance measurements. In the following (the discussion is based on our earlier work [3,4]) we will abide by this rule of general relativity, and will follow Wigner [5] in using clocks and light signals to measure space-time distances.

In Section II, we will analyze a gedanken experiment designed to measure the distance between two spatially separated points, and will show that quantum mechanics and general relativity together imply that there is a limit on the accuracy with which we can measure that distance. That uncertainty in space-time measurements is interpreted to induce an uncertainty in the space-time metrics; in other words, space-time undergoes quantum fluc-

tuations. Some consequences of space-time fluctuations are listed in Section III. Section IV is devoted to show how gravity-wave interferometers can be used to test this phenomenon of space-time fluctuations. We offer our conclusions in Section V.

II. FROM SPACE-TIME MEASUREMENTS TO SPACE-TIME FOAMS

Suppose we want to measure the distance between two separated points A and B. To do this, we put a clock (which also serves as a light-emitter and receiver) at A and a mirror at B. A light signal is sent from A to B where it is reflected to return to A. If the clock reads zero when the light signal is emitted and reads t when the signal returns to A, then the distance between A and B is given by $l = ct/2$, where c stands for the speed of light. The next question is: What is the uncertainty (or error) in the distance measurement? Since the clock at A and the mirror at B are the agents in measuring the distance, the uncertainty of distance l is given by the uncertainties in their positions. We will concentrate on the clock, expecting that the mirror contributes a comparable amount to the uncertainty in the measurement of l. Let us first recall that the clock is not stationary; its spread in speed at time zero is given by the Heisenberg uncertainty principle as

$$\delta v = \frac{\delta p}{m} \gtrsim \frac{\hbar}{2m\delta l},$$ (1)

where m is the mass of the clock. This implies an uncertainty in the distance at time t,

$$\delta l(t) = t\delta v \gtrsim \left(\frac{\hbar}{m\delta l(0)}\right)\left(\frac{l}{c}\right),$$ (2)

where we have used $t/2 = l/c$ (and we have dropped an additive term $\delta l(0)$ from the right hand side since its presence complicates the algebra but does not change any of the results). Minimizing $(\delta l(0) + \delta l(t))/2$ we get

$$\delta l^2 \gtrsim \frac{\hbar l}{mc}$$ (3)

At first sight, it appears that we can make δl, the uncertainty in the position of the clock, arbitrarily small by using a clock with a large enough (inertial) mass. But that is

wrong as the (gravitational) mass of the clock would disturb the curvature. It is here the principle of equivalence in general relativity comes into play: one cannot have a large inertial mass and a small gravitional mass since they are equal. We can now exploit this equality of the two masses to eliminate the dependence on m in the above inequality to make the uncertainty expression useful. Let the clock at A be a light-clock consisting of two parallel mirrors (each of mass $m/2$), a distance of d apart, between which bounces a beam of light. On the one hand, the clock must tick off time fast enough such that $d/c < \delta l/c$, in order that the distance uncertainty is not greater than δl. On the other hand, d is necessarily larger than the Schwarzschild radius Gm/c^2 of the mirrors (G is Newton's constant) so that the time registered by the clock can be read off at all. From these two requirements, it follows that

$$\delta l > d > \frac{Gm}{c^2},\tag{4}$$

the product of which and Eq. (3) yields [6]

$$\delta l \gtrsim (l l_P^2)^{1/3},\tag{5}$$

where $l_P = (\frac{\hbar G}{c^3})^{1/2}$ is the Planck length ($\sim 10^{-33}$ cm). In a similar way, we can deduce the uncertainty in time interval (t) measurements,

$$\delta t \gtrsim (t t_P^2)^{1/3},\tag{6}$$

where $t_P = l_P/c$ is the Planck time ($\sim 10^{-42}$ sec).

The intrinsic uncertainty in space-time measurements just described can be interpreted as inducing an intrinsic uncertainty in the space-time metric $g_{\mu\nu}$. Noting that $\delta l^2 = l^2 \delta g$ and using Eq. (5) we get

$$\delta g_{\mu\nu} \gtrsim (l_P/l)^{2/3} \sim (t_P/t)^{2/3}.\tag{7}$$

The fact that there is an uncertainty in the space-time metric means that space-time is foamy. The origin of the uncertainty is quantum mechanical. Therefore we can say that

space-time undergoes quantum fluctuations and this is an intrinsic property of space-time. The amount of fluctuations on a length scale l or time scale t is given by Eq. (7).

The uncertainty expressed in Eq. (3) is due to quantum effects, and it depends on m, the mass of the clock. In the above, we have used Eq. (4) to put a bound on m, eventually arriving at Eq. (5). Perhaps, we should point out that, besides Eq. (5), there are (at least) two other expressions for the uncertainty in space-time measurements that have appeared in the literature, predicting different degrees of foaminess in the structure of space-time. Instead of repeating the derivations used by the other workers, we find it instructive to "derive" them by adopting an argument similar to the one we have used above. We start with Eq. (3). For the bound on m, if one uses (instead of Eq. (4))

$$l \gtrsim \frac{Gm}{c^2}, \qquad (8)$$

then one finds

$$\delta l \gtrsim l_P, \qquad (9)$$

the canonical uncertainty in distance measurements widely quoted in the literature [7]. Eq. (8) gives a considerably more conservative bound on m; and the inequality is trivially satisfied because, otherwise, point B would be inside the Schwarzschild radius of the clock at A, an obviously nonsensical situation. So, we do not expect the resulting inequality (given by Eq. (9)) to be very restrictive (or, for that matter, to be very useful, in our opinion).

On the other hand, if, instead of Eq. (4), one uses

$$m_P \gtrsim m, \qquad (10)$$

where $m_P \equiv \hbar/c l_P$ denotes the Planck mass ($\sim 10^{-5}$ gm), then combining it with Eq. (3), one gets

$$\delta l \gtrsim (l l_P)^{1/2}, \qquad (11)$$

a result for the uncertainty in space-time measurements found in Ref. [8]. Since $l >> l_P$ (which we have implicitly assumed), the distance uncertainty given by Eq. (11) is considerably bigger than the one proposed by us (Eq. (5)). But regardless which of the three pictures

of space-time foam we have in mind, they all predict a very small distance uncertainty: e.g., even on the size of the whole observable universe ($\sim 10^{10}$ light-years), Eq. (9), Eq. (5), and Eq. (11) yield a fluctuation of only about 10^{-35} m, 10^{-15} m and 10^{-4} m respectively. We leave it to the readers to decide for themselves which of the three pictures of space-time foam is the most reasonable.

III. OTHER PROPERTIES OF SPACE-TIME FOAM

Let us return to that picture of space-time foam proposed by us, expressed in Eq. (5), Eq. (6), and Eq. (7). The metric fluctuations give rise to some rather interesting properties [3,4] besides the uncertainties in space-time measurements. Here is a partial list:

(i) There is a corresponding uncertainty in energy-momentum measurements for elementary particles, given by

$$\delta p \gtrsim p \left(\frac{p}{m_P c} \right)^{2/3}, \qquad \delta E \gtrsim E \left(\frac{E}{m_P c^2} \right)^{2/3}. \tag{12}$$

We should keep in mind that energy-momentum is conserved only up to this uncertainty.

(ii) Space-time fluctuations lead to decoherence phenomena. The point is that the metric fluctuation δg induces a multiplicative phase factor in the wave-function of a particle (of mass m)

$$\psi \to e^{i\delta\phi}\psi, \tag{13}$$

given by

$$\delta\phi = \frac{1}{\hbar} \int mc^2 \delta g^{00} dt. \tag{14}$$

One consequence of this additonal phase is that a point particle with mass $m > m_P$ is a classical particle (i.e., it suffices to treat it classically). This fuels the speculation that the high energy limit of quantum gravity is actually classical. But in connection with this speculation, a cautionary remark is in order: by extrapolating the mass scale beyond the

Planck mass, one runs the risk of going beyond the domain of validity of this work, viz. the low-energy limit of quantum gravity. [9]

(iii) The energy density ρ associated with the metric fluctuations (Eq. (7)) is actually very small. Regarding the metric fluctuation as a gravitational wave quantized in a spatial box of volume V, we find

$$\rho \sim m_P c^2 / V. \tag{15}$$

However, if one uses the "root mean square" approach proposed in the first paper in Ref. [6], one gets an unacceptably large energy density of $m_P c^2 / l_P^3$.

(iv) Due to space-time fluctuations, gravitational fields of individual particles with mass $m << m_P$ that make up ordinary matter are not observable. From this point of view, the gravitational field is a statistical phenomenon of bulk matter. [10]

(v) There is a simple connection between spacetime quantum fluctuations as given by Eq. (5) and the holographic principle [11]. The holographic principle asserts that the number of degrees of freedom of a region of space is bounded by the area of the region in Planck units. To see the connection, consider a region of space with linear dimension l. According to the conventional wisdom, the region can be partitioned into cubes as small as l_P^3. It follows that the number of degrees of freedom of the region is bounded by $(l/l_P)^3$, i.e., the volume of the region in Planck units. But according to our spacetime foam picture (Eq. (5)), the smallest cubes inside that region have a linear dimension of order $(l l_P^2)^{1/3}$. Accordingly, the number of degrees of freedom of the region is bounded by $[l/(l l_P^2)^{1/3}]^3$, i.e., the area of the region in Planck units, as stipulated by the holographic principle. Thus one may say that the holographic principle has its origin in the quantum fluctuations of spacetime. It has not escaped our attention that the effective dimensional reduction of the number of degrees of freedom may have a dramatic effect on the ultraviolet behaviour of a quantum field theory.

(vi) Fluctuations in space-time imply that metrics can be defined only as averages over local regions and cannot have meaning locally. This gives rise to some sort of non-locality. It has also been observed [12] that the space-time measurements described above alter the

space-time metric in a fundamental manner and that this unavoidable change in the metric destroys the commutativity (and hence locality) of position measurement operators. The gravitationally induced non-locality, in turn, suggests a modification of the fundamental commutators. Furthermore, we would not be surprised if this feature of non-locality is in some way related to the holographic principle [11].

IV. PROBING THE STRUCTURE OF SPACE-TIME WITH GRAVITY-WAVE INTERFEROMETERS

As noted above, the fluctuations that space-time undergoes are extremely small. Indeed, it is generally believed that no currently available technologies are powerful enough to probe into the space-time foam. But it has been shown [13] recently by G. Amelino-Camelia that modern gravity-wave interferometers are already sensitive enough or will soon be sensitive enough to test two of the three pictures of space-time foam described in Section II.

First let us briefly recall the physics of modern gravity-wave interferometers. They consist of a laser light source, a beam splitter, and two mirrors placed at the ends of two (very long) arms arranged in an L-shaped pattern. The light beam is split by the beam splitter into a transmitted beam and a reflected beam. The transmitted beam is directed toward one of the mirrors; and the reflected beam is directed toward the other mirror. The two beams of light are reflected by the mirrors back to the beam splitter where they are superposed. The resulting interference pattern is very sensitive to changes in the distances between the beam splitter and the mirrors at the ends of each arm. Modern gravity-wave interferometers are sensitive to changes in distance to an accuracy of the order of $10^{-18}m$ and better. To reach such a sensitivity, one has to contend with all sorts of noises such as seismic noise, suspension thermal noise, and photon shot noise. Our claim is that even after one has subtracted away all these known noises, there is still a noise arising from space-time fluctuations.

At first sight, it appears that the task of measuring space-time fluctuations is well beyond

our reach; after all, even the extraordinary sensitivity down to an accuracy of order 10^{-18} m is no where near the Planck scale of 10^{-35} m. But the displacement sensitivity of an interferometer actually depends on frequencies f (more on this below). Besides the 10^{-18} m length scale mentioned above, the physics of interferometers involves another length scale c/f provided by f. Interestingly, as shown in Ref. [13], within certain range of frequencies, the experimental limits are comparable to the theoretical predictions for two of the space-time foam pictures described above.

The idea of using gravity-wave interferometers to probe the structure of space-time is actually fairly simple. Let us concentrate on the picture of space-time foam described by Eq. (7) and Eq. (5) or Eq. (6). Due to the foaminess of space-time, in any distance measurement that involves an amount of time t, there is a minute uncertainty $\delta l \sim (ctl_{QG}^2)^{1/3}$, where, for later use, we have introduced l_{QG} which we expect to be of order l_P. (It is understood that the time of observation t is much smaller than the time interval over which the space-time region where the observation is done experiences significant curvature effects.) But measuring minute changes in (the) relative distances (of the test masses or the mirrors) is exactly what an interferometer is designed to do. Hence, the intrinsic uncertainty in a distance measurement for a time t manifests itself as a displacement noise (in addition to other sources of noises) that infests the interferometers

$$\sigma \sim (ctl_{QG}^2)^{1/3}. \tag{16}$$

In other words, quantum space-time effects provide another source of noise in the interferometers and that noise is given by Eq. (16). It is customary to write the displacement noise in terms of the associated displacement amplitude spectral density $S(f)$ of frequency f. For a frequency-band limited from below by the time of observation t, σ is given in terms of $S(f)$ by [14]

$$\sigma^2 = \int_{1/t}^{f_{max}} [S(f)]^2 df. \tag{17}$$

Now we can easily check that, for the displacement noise given by Eq. (16) corresponding to our picture of space-time foam, the associated $S(f)$ is

$$S(f) \sim f^{-5/6}(cl_{QG}^2)^{1/3}. \tag{18}$$

In passing, we should mention that since we are considering a time scale much larger than the Planck time, we expect this formula for $S(f)$ to hold only for frequencies much smaller than the Planck frequency (c/l_P). For consistency, this implies that if the $S(f)$ given by Eq. (18) is used in the integral in Eq. (17), the integral should be relatively insensitive to f_{max}. That is indeed the case as the small frequency region dominates the integral for σ. Needless to say, to know the high frequency behavior of $S(f)$, one would need the correct theory of quantum gravity.

We can now use the existing noise-level data [15] obtained at the Caltech 40-meter interferometer to put a bound on l_{QG}. In particular, by comparing Eq. (18) with the observed noise level of 3×10^{-19}mHz$^{-1/2}$ near 450 Hz, which is the lowest noise level reached by the interferometer, we obtain the bound $l_{QG} \lesssim 10^{-29}$ m which is in accordance with our expectation $l_{QG} \sim l_P \sim 10^{-35}$ m. The exciting news is that the "advanced phase" of LIGO [16] is expected to achieve a displacement noise level of less than 10^{-20}mHz$^{-1/2}$ near 100 Hz, and this would probe l_{QG} down to 10^{-33} m which is almost the length scale that we expect it to be. Moreover, since $S(f)$ goes like $f^{-5/6}$ according to Eq. (18), we can look forward to the post-LIGO/VIRGO generation of gravity-wave interferometers for improvement by optimizing the performance at low frequencies. As lower frequency detection is possible only in space, we will probably need to wait for a decade or two for the LISA-type set-ups [17]; but it will be worth the wait!

We can also test the other two pictures of space-time foam by using the gravity-wave interferometers. The results [18] are shown in the accompanying Table where, for convenience, we have rewritten Eq. (9) and Eq. (11) respectively as $\delta l \gtrsim L_{QG}$ and $\delta l \gtrsim (l\tilde{l}_{QG})^{1/2}$. We expect both L_{QG} and $\tilde{l}_{QG}$ to be of order $l_P \sim 10^{-35}$ m. Note that the amplitude spectral density for each of the three space-time foam pictures has its own characteristic frequency

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dependence.

Spacetime pictures with $\delta l \gtrsim$	L_{QG}	$(ll_{QG}^2)^{1/3}$	$(l\tilde{l}_{QG})^{1/2}$
Metric fluctuations with $\delta g \gtrsim$	$\frac{L_{QG}}{l}$	$(\frac{l_{QG}}{l})^{2/3}$	$(\frac{\tilde{l}_{QG}}{l})^{1/2}$
Displacement noise σ	L_{QG}	$(ctl_{QG}^2)^{1/3}$	$(ct\tilde{l}_{QG})^{1/2}$
Amplitude spectral density $S(f)$	$f^{-1/2}L_{QG}$	$f^{-5/6}(cl_{QG}^2)^{1/3}$	$f^{-1}(c\tilde{l}_{QG})^{1/2}$
Bound from 40-m interferometer	$L_{QG} \lesssim 10^{-17}$ m	$l_{QG} \lesssim 10^{-29}$ m	$\tilde{l}_{QG} \lesssim 10^{-40}$ m
Advanced phase of LIGO probes	L_{QG} to 10^{-19} m	l_{QG} to 10^{-33} m	$\tilde{l}_{QG}$ to 10^{-45} m
Present status	hard to check	waiting eagerly	ruled out?

V. CONCLUSIONS

As the last column of the accompanying Table shows, the existing noise-level data obtained at the Caltech 40-m interferometer have already excluded all values of $\tilde{l}_{QG}$ down to 10^{-40} m, five orders of magnitude smaller than the Planck length. Thus, the third picture of space-time foam appears to be in serious trouble, if not aleady ruled out. It is interesting to reflect that, until recently, no one would have dreamed that there is a way to rule out a space-time foam model that predicts a mere 10^{-4} m uncertainty on a scale of the whole observable universe. Now, even the Planck scale is no longer regarded as so prohibitively small that quantum gravity cannot be probed by modern laser interferometry.

On the other hand, the Table also shows that the quantum space-time effects predicted by the canonical picture of space-time foam (corresponding to fluctuations given by Eq. (9) in space-time measurements) are still far too small to be measured by interferometry technologies currently available or imaginable. Even the advanced phase of LIGO can probe L_{QG} only down to 10^{-19} m, some 16 orders away from the expected scale of Planck length. Waiting for the confirmation of the canonical space-time foam picture with the techniques of interferometry is like waiting for Godot in Beckett's play — the waiting may never end.

Finally, here is the exciting news: modern gravity-wave interferometers are within striking distance of testing the space-time foam picture proposed by us [3,4,6]. Incredibly, the

advanced phase of LIGO will probe l_{QG} down to 10^{-33} m. We can expect even more stringent bounds on l_{QG} with future LISA-type projects. [17] According to our space-time foam picture, a noise-level corresponding to the associated amplitude spectral density given by Eq. (18) with l_{QG} of the order of Planck length, should be left in the read-out of an interferometer even after all classical-physics and ordinary quantum-mechanics noise sources have been eliminated. That noise is an intrinsic consequence of quantum gravity. If and when that noise is detected, we will have successfully taken a glimpse at the very fabric of space-time at very short distance scales. Eagerly we wait to catch that faint echo from space-time quantum fluctuations.

Acknowledgments

One of us (YJN) gave a seminar on the topic of space-time measurements to a very receptive audience at the University of Connecticut in the fall of 1993. In the audience was Prof. Kurt Haller who probably did raise the question: How can we test the uncertainty expressed in Eq. (5)? At that time we had no concrete and practical idea. Now, five and half years later, we are glad to report to Prof. Haller that there is a way to do it. This article is dedicated to him to celebrate his seventieth birthday.

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